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AN EVALUATION OF TRACER DILUTION TECHNIQUES FOR GAUGING OF RIVERS IN FLOOD

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ABSTRACT

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The use of the tracer dilution technique to gauge flow over broad shallow floodplains is examined. Because of the long mixing lengths, it sometimes takes several days for the passage of the laterally dispersed pulse. Tracer methods can be used if the flow rates vary linearly during the passage of the pulse. The measured flow rate is related to the time at which the first moment of the concentration profile ($\int tc(z,t)\mathrm{d}t$) is zero. An experimental verification is presented.

By analysing the tracer pulse shapes before the establishment of complete mixing, it was demonstrated that the effective dispersion coefficients were independent of the scale of turbulence over the range 10 m to ~ 1 km. This is consistent with the establishment of isotropic turbulence on the floodplain in contrast to oceanic surfaces. The velocity of the tracer is a factor of 2 less than that of an advancing wave front, which is in acceptable agreement with prediction.

It is concluded that the transport of a non-interacting contaminant across the floodplain can be predicted from the wave front velocity and the dispersion coefficients measured close to the release point.

INTRODUCTION

Classic dilution techniques for river gauging have the advantage that, provided complete mixing has been achieved, discharge measurements can be made from observations at a single point and that information on cross-sections is not required (Hull, 1958; Clayton and Smith, 1963). The experimental and theoretical bases of the methods have been developed for flow in pipes, open channels and small turbulent rivers (Ellis, 1967; Dinçer, 1967), and their validity for large discharges has been confirmed (Florkowski et al., 1969; Cless Berenet et al., 1970). Special considerations arise when attempting to gauge flow across broad shallow floodplains. Three aspects of this problem are discussed: (1) the approach to isotropic turbulence; (2) the use of tracer dilution techniques to gauge rivers during periods of varying discharge; and (3) the relative transport rates of a wave front and a tracer pulse.

Aspect (2) is of particular practical importance on many floodplains where the time of passage of fully mixed pulses is long. The validity of the tracer dilution technique was experimentally demonstrated on the lower reaches of the flooded Magela Creek (Fig. 1). The leading edge of the tritium pulse was observed 8 days after the commencement of injection and the duration of the plume was ~ 9 days. During the passage of the tracer, the discharge estimated by conventional techniques at the Northern Territory Water Division's gauging station opposite the observation transect, decreased regularly from 90 to 45 $m^3 s^{-1}$. It is shown theoretically that the time to which the flow-rate measurement refers is that at which the first moment of the tracer concentration profile ($\int_0^\infty tc(z,t)dt$) is zero.

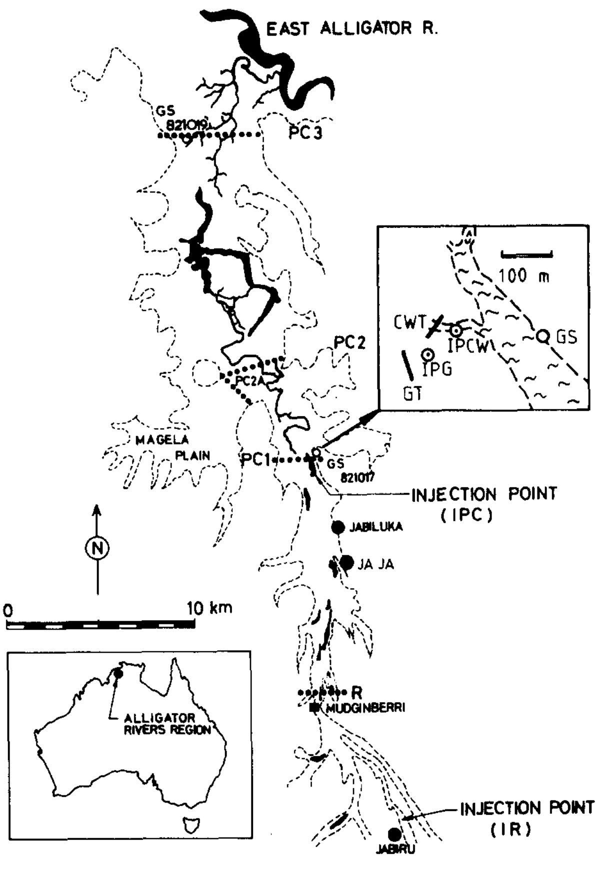

Fig. 1. Sketch of the experimental area showing the injection points IR and IPC, the measuring transects R, PC*1*, PC*2*, PC*3* and the gauging stations (GS) *821017* and *821019*. The insert shows the location of the transects CWT and GT and the associated injection points IPCW and IPG.

METHODS

The Magela Creek

The Magela Creek is a tributary of the East Alligator River in the Alligator Rivers region of the Northern Territory of Australia (Fig. 1). It has a catchment area of $\sim 1560\,km^2$ and a mean annual discharge, based upon recorded flows, of $1650 \cdot 10^6\ m^3$. The Magela is located in an area which experiences a monsoonal climate with distinct wet and dry seasons; 97% of the annual rainfall occurs in the wet season from October to May. Average rainfall for the region, based on long-term regional and short-term local data, is 1530 mm. Evaporation is also high; the average potential evaporation exceeds rainfall by ~ 700 mm.

During the wet season, the Magela floodplain is a continuous body of water ~ 2—5 m deep, up to 15 km wide, and extending 30 km. However, the net evaporation of water creates a series of discrete billabongs of varying sizes and characteristics throughout the area during the dry season. In the headwaters zone, there are numerous waterfalls and rapids where stream velocities under flood conditions can be high. In the middle reaches, velocities are lower but in the main streams they exceed $1\,m\,s^{-1}$ under flood conditions. On the floodplain, velocities are low and at expected annual return period flood levels they range from < 0.1 to $0.2\,m\,s^{-1}$ in the main billabongs and over most of the clear water sheet-flow zones.

In the last decade interest in the characteristics of the Magela Creek have been associated principally with the discovery of two major uranium ore bodies — Ranger and Jabiluka — in areas adjacent to the floodplain. Much investigational work has been undertaken to describe the Magela and considerable detail is available elsewhere (Christian and Aldrick, 1977; Morley, 1981).

Experimental technique

Acidified solutions of tritium labelled water HTO (3700 GBq) and ^{65}Zn (30 GBq) were injected into the creek at a constant rate over 33 hr. The duration of the release was chosen to ensure the establishment of a quasi-steady state and to enable sufficient time to study the distribution of zinc during the passage of the pulse. As this paper is concerned with the problem of gauging on floodplains, only the tritium data are discussed. The locations of the injection points, and the observation transects PC*1*, PC*2* and PC*3* are shown in Fig. 1. Two separate injections were made, the first at IPC (February 19—20, 1978) and the second at IR (March 2—3, 1978). The transects were defined at regular intervals. The tritium levels were determined by mixing 10-ml aliquots of the water with 11 ml of the emulsion scintillating agent Instagel® in polythene vials and counting them in a Packard® model *3255* liquid scintillation spectrometer. All samples were compared against the tritiated water standard NBS *4926*. The mean background was 6.0 counts $min.^{-1}$; the counter efficiency was 19.8%.

RESULTS

Tritium release from injection point IPC

The tracer was released at a steady rate between $07^h 00^m$ on February 19, 1978 and $15^h 00^m$ on the following day. The daily rainfall at Ja Ja and the daily discharge rates of the Magela system at gauging stations *821017* and *821019* (Fig. 1) during the investigation are shown in Fig. 2. The traverse times of the tritium pulse at the nearby observation transects PC*1* and PC*3* are also shown.

Assuming that the discharge maximum A (Fig. 2b) can be correlated with that at A' (Fig. 2c), the ratio of the rate of transport of the wave crest to that of the tritium pulse centroid is ~ 2:1.

The segmented transect PC2—PC2A was established to monitor not only the downstream movement of the labelled water, but also its transport across the entrance of the large western embayment of the Magela Plain. The data are synthesised as an artificial pulse in Fig. 3B. This figure simply indicates the variation of tritium with time over the two segments, but does not imply that the angle of approach of the pulse centroid is known. The ratio of tritium intersecting the PC2 and PC2A segments is 2.1:1.

The tritium profile across transect PC*3* is shown in Fig. 3A. The general shape can best be interpreted as the coalescence of two pulse segments which diverge in the general vicinity of transect PC2—PC2A. Evidence to support this hypothesis is two-fold:

(1) The ratio of the total tritium content in the eastern to that in the western lobe of the pulse (2.2:1), is similar to that between PC2 and PC2A.

(2) The delayed arrival of the centroid of the western lobe of the pulse is consistent with the longer flow path of water transported across PC2A.

Clearly, the condition of complete mixing has not been achieved, and hence the data cannot be used for stream gauging. However, estimates of the transverse dispersivities can be obtained from an analysis of the pulse shape.

Tritium release from injection point IR

The tracer was released between $07^h 00^m$ on March 2, 1978 and $16^h 00^m$ March 3, 1978, and the resulting profiles were observed at transects R and PC*1* (Fig. 1). In both cases, the degree of lateral mixing was sufficient to enable stream gauging by dilution techniques. A detailed analysis was made of the data from the latter transect, which was established opposite gauging station *821017* (Fig. 1) by the Northern Territory Department of Transport and Works (Water Division); an independent rating curve is available.

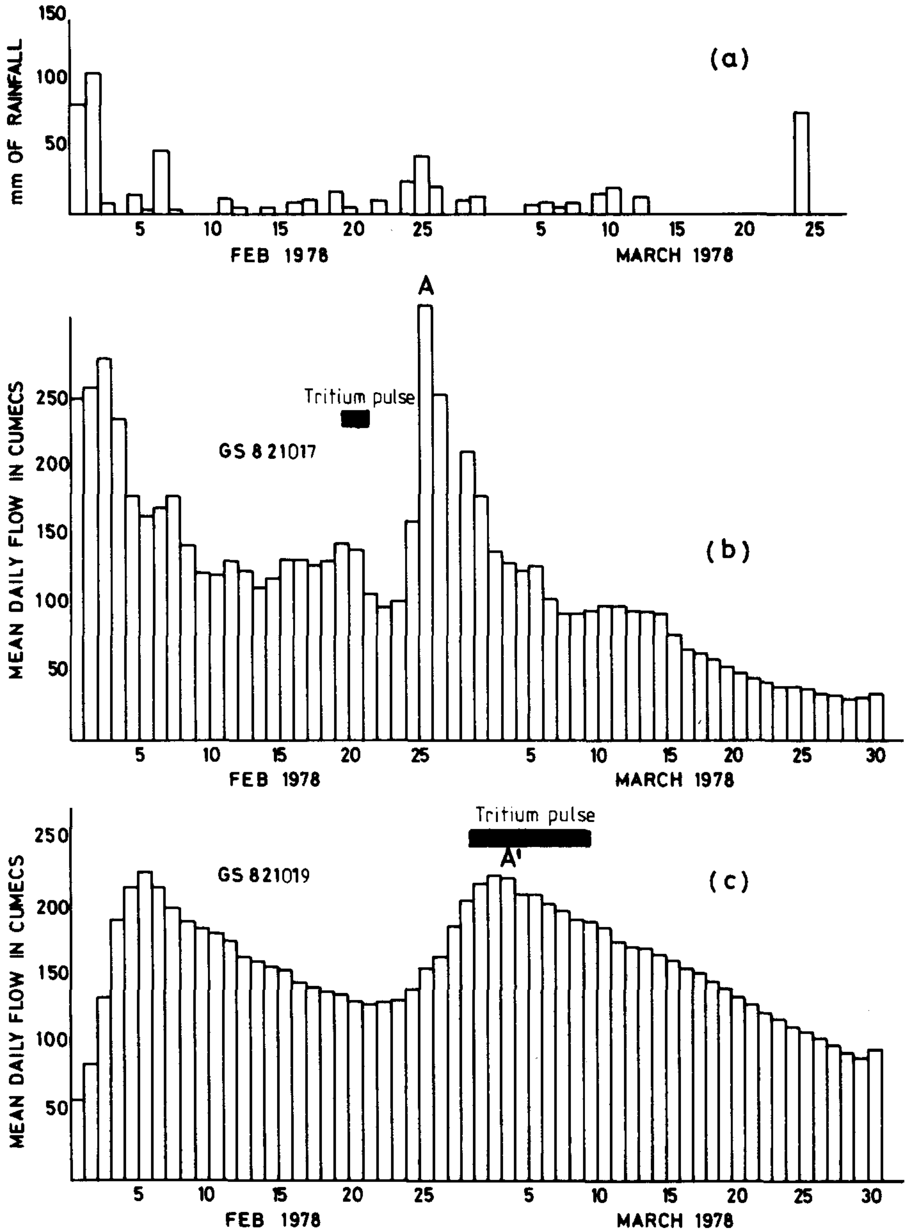

Fig. 2. The daily rainfall at Ja Ja (a), and the daily discharges at the gauging stations (GS) *821017* (b) (at transect PC*1*), and *821019* (c) (at transect PC*3*). The tritium pulses (full width half maximum) at the two observation points following the injection at IPC are shown.

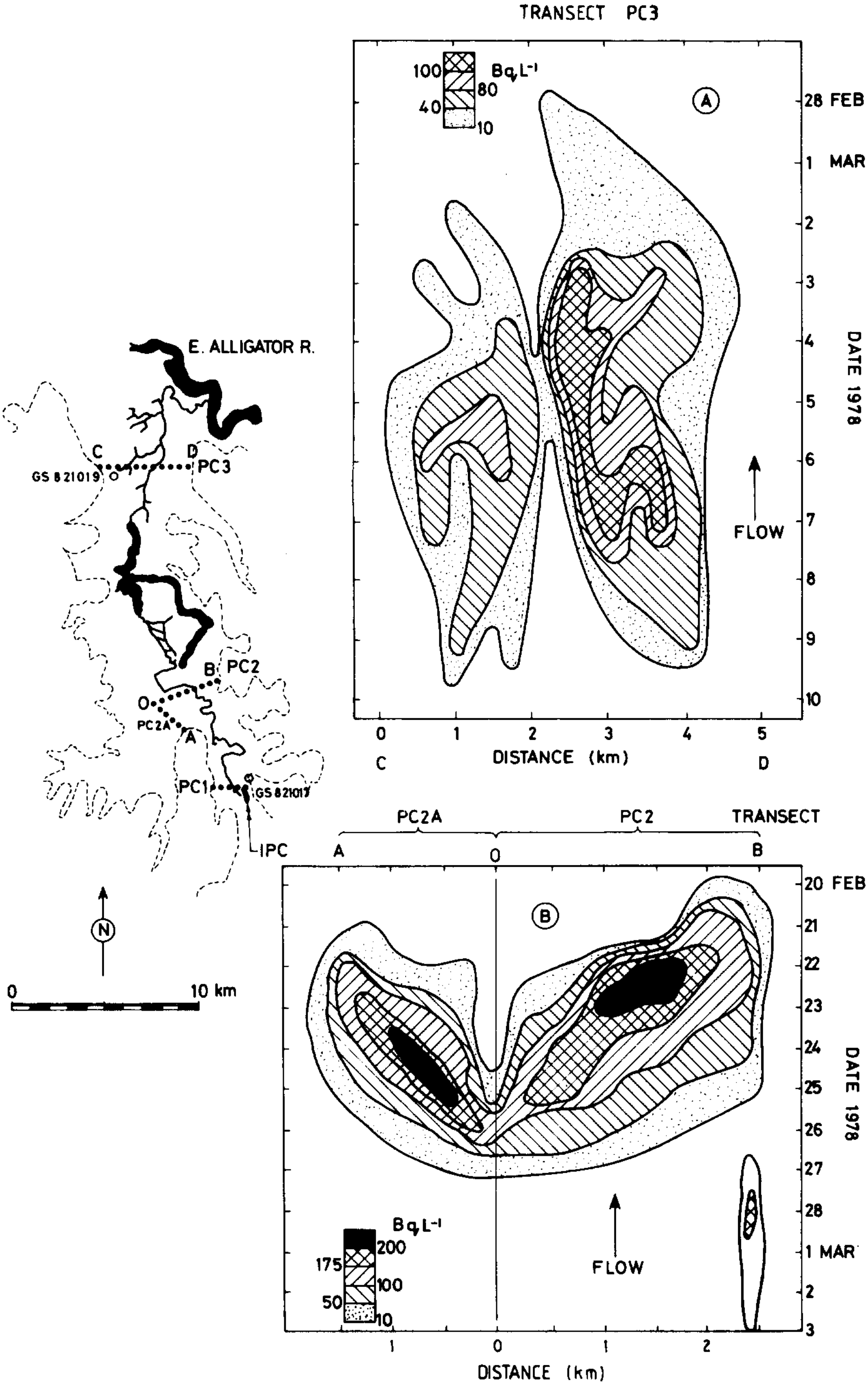

Fig. 3. The tritium pulse at transect PC*3* (A) and at transect PC*2*—PC*2*A (B), following injection at IPC.

Tracer dispersion on the floodplain

The radioactive tracer $^{99m}TcO_4^-$ was released at a steady rate at two additional points on the floodplain near transect PC*1* — one in a vegetated area and one in a clear area. The activity profiles were measured along temporary transects GT and CWT (Fig. 1) 53 m from these injection points. The data which are shown in Fig. 4a and b were used to calculate the solute dispersivities in the vicinity of the release sites.

DISCUSSION

Scale of turbulence on the Magela floodplain

A turbulent stream may be considered as a spectrum of eddies superimposed on the general flow direction. In his classic work on momentum transfer, L. Prandtl developed the concept of the characteristic eddy length, l_P, which is a measure of the scale of the eddies and analogous to the mean free path of molecules in the kinetic theory of gases. For open-channel flow, l_P can be calculated from the semi-empirical formula (Henderson, 1966):

$$l_P = \kappa y (1 - y/y')^{1/2} \quad (1)$$

where κ is von Karman's universal constant with the value of 0.4; y is the distance from the solid boundary; and y' is the total depth. Provided the conditions of isotropic turbulence on the floodplain (depth 2 m) are approached, the Prandtl mixing length will be of the order of a few metres. On the other hand, if the turbulence were not isotropic, and the scale of the largest eddies is determined principally by the distance between the banks ($\gtrsim$ 5 km), the transverse (z) component of the mixing length will be substantially greater.

The two hypotheses may be distinguished by studying the variation with distance of the eddy dispersion coefficients calculated from the Gaussian approximation to the solution of the transport equation.

For continuous injection of a tracer, it takes the form (Petersen, 1977):

$$c(x_0,z) = [Q/(4\pi v D_E^z x_0)^{1/2}] \exp(-vz^2/4D_E^z x_0) \quad (2)$$

where $c(x_0,z)$ is the tracer concentration at the observation plane x_0; Q is the rate of injection of the tracer; v is the mean flow velocity; and D_E^z and D_E^z/v are eddy dispersion coefficients and dispersivities in the z-direction, respectively.

Dispersivities were measured over distances ranging from 53 m to 21 km from the injection points. As indicated in the experimental section, concentration profiles at the 53-m transects CWT and GT (Fig. 1) are shown in Fig. 4a and b. The solid curves represent eq. 2 with dispersivities of 0.15

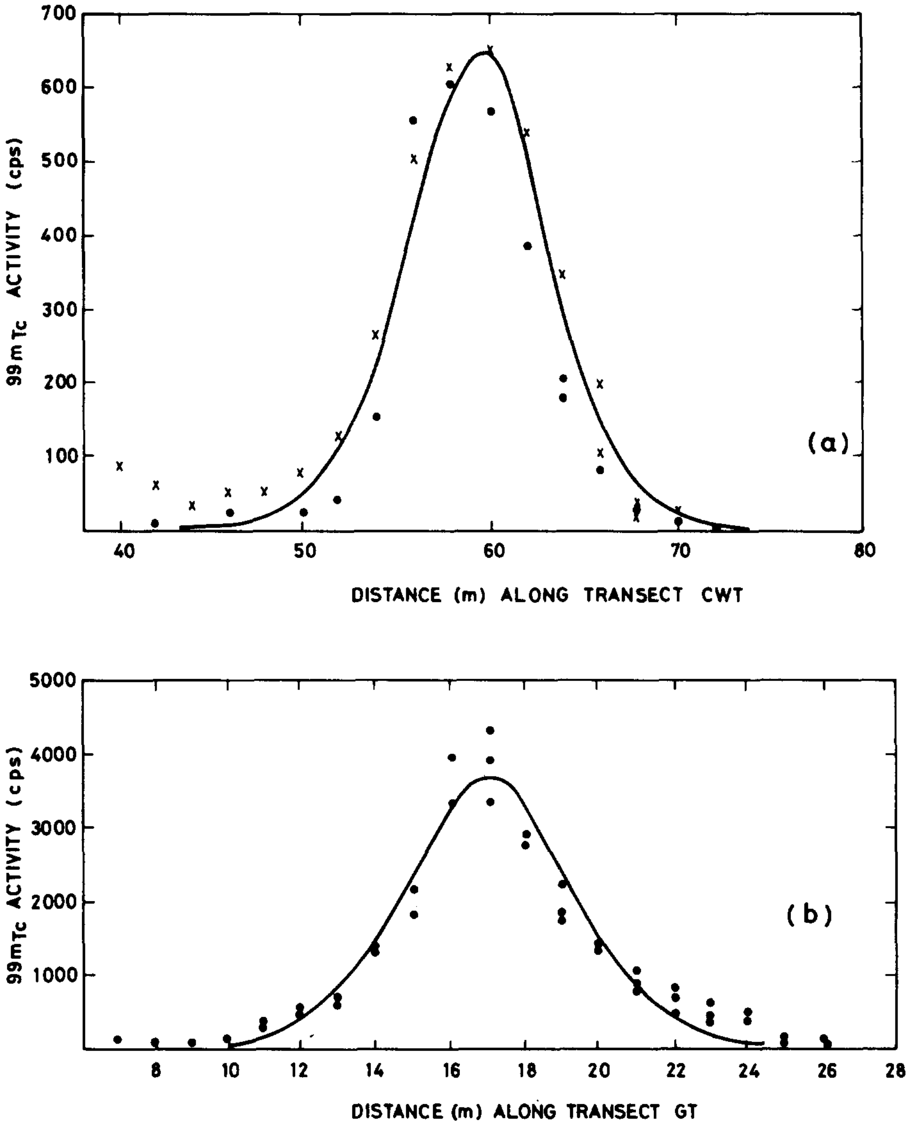

Fig. 4. ^{99m}Tc profile following steady release at IPCW and IPC 53 m upstream of the transects CWT (a) and GT (b) (Fig. 1, insert).

and 0.04 m, respectively. The differences may be due to the effects of vegetation on solute dispersion. In both cases, the depth was 0.5 m. Similar estimates were made of the solute dispersivities at PC2 and PC3 (eastern lobe of the pulse, Fig. 3) which are 7 and 21 km from the injection point IPC (Fig. 1), to yield experimental values of 0.05 to 0.02 m, respectively.

Since, experimentally, the average linear flow velocity (calculated from the rate of transport of the pulse centroid) varied by less than a factor of 2, the measured dispersivities are an adequate indicator of the eddy dispersion coefficients. The data are precise enough to enable an important distinction to be made between dispersion on floodplains and lateral dispersion on oceanic surfaces.

There is a substantial body of experimental data showing that for releases in the mixed layer above the oceanic thermocline, the dispersion coefficient increases without limit with the magnitude of the pulse. Okubo (1971) has assessed data from a large number of instantaneous releases and found that the horizontal eddy dispersion coefficient K_a ($cm^2\ s^{-1}$) increases as the 1.15 power of the horizontal scale l (cm). The parameter l is defined by $3\sigma_{rc}$, where σ_{rc}^2 is the variance of the radially distributed pulse.

By contrast, there is no experimental evidence on the floodplain for an increase in the dispersion coefficient with distance downstream of the release point, even though the length scale measured by (transverse variance)$^{1/2}$ increased by a factor greater than 300. It therefore follows that the characteristic eddy length is substantially less than the pulse width which, in one case (Fig. 4b), was ~ 5 m. This conclusion is further confirmed by the fact that at a distance of 53 m from the injection point, the concentration profiles (Fig. 4) are Gaussian as predicted by the *distant field* approximation to the solution of the transport equation.

The data are consistent with the hypothesis that a condition of isotropic turbulence is approached on a broad floodplain, and that the characteristic eddy length l_P is given by eq. 1.

River discharge measurements (constant flow)

Provided that there are no sources or sinks of tracer, the equation of continuity can be expressed in the form:

$$\Delta \cdot CV = -\partial C/\partial t \tag{3}$$

where V is the mean flow velocity; and C the tracer concentration. Evans and Ely (1964) showed from eq. 3, that:

$$\int_t \int_v \nabla \cdot (CV) \mathrm{d}\nu \mathrm{d}t = A \tag{4}$$

where A is the total activity of the tracer; and the integration is over any finite volume ν containing all the tracer at $t = 0$, and over time between $t = 0$ and $t = T$ when all the tracer has left the volume.

Eq. 2 may be expressed in the form:

$$A = \int_t \int_z c(z,t) q(z,t) \mathrm{d}z \mathrm{d}t \tag{5}$$

where $q(z,t)\mathrm{d}z$ is the volume flow rate between z and $(z + \mathrm{d}z)$ under the following conditions:

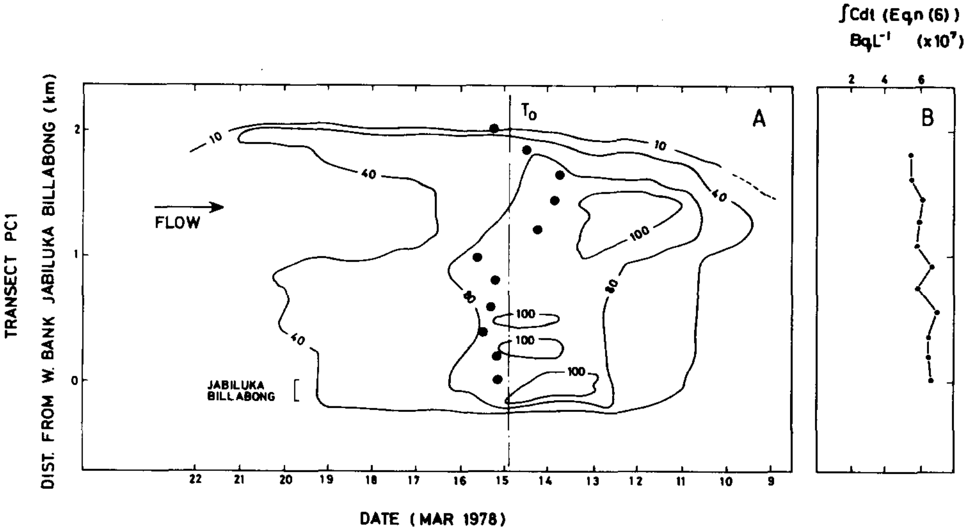

Fig. 5. Tritium profile (Bq l^{-1}) at transect PC*1* following the release of 3700 GBq of tritium at a constant rate at IR. The integrated concentration showing the degree of mixing is also shown. The time T_0 is the time at which the estimate of the discharge refers. The estimates of T_0 which would have been made at individual points are also shown (•).

(1) There is no net component of flow normal to the principal flow direction. This implies that, as was shown in the previous section, the distance between injection and observation is large compared with the eddy diffusion length.

(2) The tracer concentration is independent of depth y on the shallow floodplain.

If the tracer concentrations monitored at a single point z are to be interpreted as a volume flow rate Q, it is necessary to separate the integral (5). In the first instance, it is assumed that discharge Q is independent of time, i.e. $q(z,t) = q(z)$. Separability of the integral is possible under either of the following circumstances:

(a):

$$c(z,t) = \mathrm{Co} \quad \text{(a constant)} \tag{6}$$

Eq. 6 implies that the tracer is injected at a constant rate and complete mixing is achieved.

(b) The integral:

$$\int_0^\infty c(z,t)\mathrm{d}t = F \tag{7}$$

is independent of z. Eq. 5 could then be simplified to:

$$A = F\int_z q(z)\mathrm{d}z = FQ \tag{8}$$

where the discharge Q (a constant) $= \int_z q(z)\mathrm{d}z$.

Airey (1983) has shown that eq. 7 is the criterion for complete mixing for a generalized tracer input function. The validity of eq. 7 at transect PC*1* is demonstrated in Fig. 5B. The value of the integral was obtained by summing measured concentrations of tritium in samples collected at regular intervals at eleven sampling points across the transect.

The constancy of integral (7) is experimental evidence for the condition of complete mixing. However, eq. 8 cannot be used without further refinement for estimating Q if the discharge across the floodplain varied appreciably during the extended period of the passage of the pulse.

Stream gauging under conditions of variable flow

Because of the breadth of many floodplains, substantial time is required to achieve complete mixing. At transect PC*1* for instance, the leading edge of the 11-day pulse arrived ~ 8 days after the injection. Substantial changes in discharge during the passage of the pulse were observed (Fig. 2). Such complications are fairly common and apply also, for example, to mountain streams which rise and fall rapidly.

Following Guizerix et al. (1970) it has been shown (Airey, 1983) that under conditions of complete mixing:

$$\mathrm{d}A/\mathrm{d}t = Q(t)c(t) \tag{9}$$

where $\mathrm{d}A/\mathrm{d}t$ is the constant rate of tracer injection; $Q(t)$ is the variable discharge; and $c(t)$ is the tracer concentration which will be independent of the position z across the transect.

Continuous injection is clearly impractical under most conditions. It is therefore necessary to examine the effect of variable flow on the response to a generalized tracer input function. In the following treatment it will be assumed that the discharge varies linearly with time but, as a consequence, there is no change in flow pattern, i.e.:

$$q(z,t) = q_0(z,t_0)(1 + \alpha t) \tag{10}$$

where $q(z,t)$ and $q_0(z,t_0)$ is the rate of flow between z and $(z + \mathrm{d}z)$ at times t and t_0, respectively.

From eqs. 8 and 10:

$$A = Q_0F + \alpha\int_z q_0(z,t_0)\left[\int_t tc(z,t)\mathrm{d}t\right]\mathrm{d}z \tag{11}$$

where

$$Q_0 = \int_z q_0(z,t)\,\mathrm{d}z \tag{12}$$

Eq. 11 reduces to the simpler form:

$$A = Q_0 F \tag{13}$$

when the integral of the first moment of the concentration:

$$\int_0^\infty tc(z,t)\mathrm{d}t = 0 \tag{14}$$

Under these conditions the discharge Q_0 which is valid for a time T_0, is calculated from the total injected activity A, and the parameter F (eq. 7) which is the total integrated concentration on the total integrated count rate, over the pulse, in the appropriate units.

The time T_0 at which eq. 14 is valid can be determined numerically provided that information on the complete pulse shape is available. Under many circumstances, it is not possible to monitor across the transect, and consequently an error is introduced in the estimate of T_0. Apparent values of T_0 obtained from an evaluation of the integral (14) at specific points on the transect PC*1* are plotted in Fig. 3c. The "true" value obtained from the total integral is also shown.

In the present experiment, 100 Ci (3700 GBq) of tritium was injected. The rate of flow Q_0, calculated from eq. 13 was 53 $m^3 s^{-1}$. The time at which measurement was valid, i.e. eq. 14 was satisfied, was March 14. The results of conventional gauging at PC*1* are shown in Fig. 2. During the course of the experiment, the discharge fell from ~ 90 to 45 $m^3 s^{-1}$ (Water Division, pers. commun., 1978). On the day in question the "conventional" flow rate was 70 $m^3 s^{-1}$. Given the difficulties inherent in gauging flood-plains both by conventional and by tracer dilution techniques, the agreement is considered satisfactory.

The relationship between the water speed and wave speed

In the introductory section it was shown that the ratio of the rate of transport of the wave crest is twice that of the tritium pulse centroid between the transects PC*1* and PC2. If the wave behaves as a monoclinal rising flood wave travelling at a constant velocity, it follows from the continuity relationship (Walshaw and Jobson, 1972) that the wave speed V_w can be expressed by:

$$V_w = \mathrm{d}Q/\mathrm{d}A \tag{15}$$

where Q is the discharge; and A the cross-sectional area of flow. Brady and Johnson (1980) have pointed out that by combining eq. 15 (the Kleitz—Seddon relationship) with the Manning formula for a wide rectangular channel:

$$V_w/V = 1.67 \tag{16}$$

They tested this expression for three reaches of the River Wear (England) at low- and high-flow conditions. At low discharge V_w/V ranged between 2.66

and 2.97, and at high discharge between 1.67 and 2.18. Brady and Johnson (1980) attribute the better agreement in the latter case to the fact that high discharges are instrumental in carving out the stream bed and therefore exist in a morphological environment which more clearly conforms to the natural hydraulic conditions.

There are obvious difficulties in applying eq. 16 to a complex floodplain with important inflow and backflow regions. The experimentally observed value of V_w/V (2.0) is therefore in acceptable agreement with that predicted. The only conclusion that can be drawn is that a combination of the Manning and Kleitz—Seddon (eq. 15) formulae can be used to predict the relative retardation of the solute and wave front under the experimental conditions. However, insufficient information is available to test the two equations independently.

CONCLUSIONS

An assessment was made of the application of tracer dilution techniques to the gauging of flow over broad shallow floodplains. The following conclusions are drawn:

(1) Because of the breadth of the floodplains and the resulting long mixing lengths, the time for the passage of the tracer pulse may be of the order of days. The tracer dilution techniques can be used directly if the discharge varies linearly during the observation of the tracer profile. The calculated flow rate refers to the time at which the first moment of the concentration profile ($\int_0^\infty tc(z,t)\mathrm{d}t$) is zero. Ideally, the integral should be evaluated over the complete pulse. However, an estimate can be made by averaging data obtained from specific points across the transect.

(2) The lateral dispersion coefficients are essentially independent of the size of the pulse. This is in contrast to the behaviour of radial dispersion coefficients associated with surface oceanic dispersion which appear to increase without limit. It is thus concluded that conditions of isotropic turbulence are approached on floodplains with a Prandtl mixing length determined principally by the depth.

(3) Under the experimental conditions the velocity of the tritium pulse, relative to that of an advancing wave front, was retarded by a factor of 2. This is in acceptable agreement with the theoretical value of 1.67. From this, and from the previous result it is concluded that the transport of non-interacting contaminants on floodplains can be predicted from the velocity of wave fronts and the dispersion coefficients close to the release point.

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